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06MAT11

First Semester B.E. Degree Examination, December 2012
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose your answers for the following : **(04 Marks)**

i) If $y = \sin^2 x$, then y_n is

A) $-2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$ B) $2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$

C) $2^{n-1}\sin\left(2x+\frac{n\pi}{2}\right)$ D) $-2^{n-1}\sin\left(2x+\frac{n\pi}{2}\right)$

ii) If $y = x\log(x+1)$ then y_n is

A) $\frac{(-1)^{n-1}(n-1)!x}{(x+1)^{n+1}}$ B) $\frac{(-1)^{n-1}(n-2)!(x+n)}{(x+1)^{n}}$

C) $\frac{(-1)^{n-1}(n-2)!(x+n)}{(x+1)}$ D) None of these.

iii) The angle of intersection of the curves $r=\frac{a\theta}{1+\theta}$, $r=\frac{a}{1+\theta^2}$ is

A) $\cos^{-1}3$ B) $\cot^{-1}3$ C) $\tan^{-1}\frac{1}{3}$ D) $\tan^{-1}3$.

iv) Pedal equation of the curve $r^m\cos m\theta = a^m$ is

A) $r^{m-1}=a^m$ B) $p^2=a^m r^{m-1}$ C) $pr^{m-1}=a^m$ D) $p^2=r^m a^m$

b. Find y_n, if $y=e^{-3x}\cos^3 x$ **(04 Marks)**

c. If $y^{1/m}+y^{-1/m}=2x$, prove that $(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2+m^2)y_n=0$ **(06 Marks)**

d. Find the angle between the curves $r = a\log\theta$, $r = a/\log\theta$. **(06 Marks)**

2 a. Choose your answers for the following : **(04 Marks)**

i) If $u=\frac{x^2}{y}$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to

A) 2u B) u C) 0 D) 1

ii) If u is a homogeneous function of order n is x&y then $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ is

A) nu B) n^2u C) $n(n-1)u$ D) $n(n+1)u$.

iii) If $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is equal to

A) 1 B) r C) $\frac{1}{r}$ D) 0

iv) $\frac{\delta x}{x}$ is called

A) Absolute error B) Relative error C) Percentage error D) Absolute & relative error.

b. If $u = \operatorname{cosec}^{-1}\left[\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right]^{1/2}$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}-\frac{\tan u}{12}\left[\frac{13}{12}+\frac{\tan^2 u}{12}\right]$$ **(04 Marks)**

c. If $u=\frac{yz}{x}$, $v=\frac{zx}{y}$, $w=\frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{0(x,y,z)}=4$ **(06 Marks)**

d. If the H.P. required to propel a steamer varies as the cube of the velocity and square of the length prove that a 3% increase in velocity and 4% increase in lengths will require an increase about 17% H.P. **(06 Marks)**

3 a. Choose your answers for the following : **(04 Marks)**

i) $\int_0^{\pi}\sin^7 x\,dx$ is equal to

A) zero B) $\frac{32\pi}{35}$ C) $\frac{32}{35}$ D) $\frac{35\pi}{35}$

ii) $\int_0^{\infty}\frac{dx}{(1+x^2)^{7/2}}$ is equal to

A) $\frac{8}{15}\pi$ B) $\frac{8}{15}$ C) $\frac{4}{15}$ D) $\frac{15}{8}$

iii) The shape of the curve $r^2 = a^2\cos 2\theta$ is

A) Three leaved B) cycloid C) cardiode D) Lemnisate of Bernoulli

iv) The curve $y^2(a-x) = x^2(a+x)$ passes through

A) origin B) Node C) x-axis D) y-axis

b. If $I_{m,n} = \int_0^{\pi/2}\sin^m x\cos^n x\,dx$ ($m > 0, n > 0$) ; show that $I_{m,n} = \frac{n-1}{m+n}\ I_{m,n-2}$ **(04 Marks)**

c. Evaluate $\int_{\theta}^{a}(x^2+a^2)^{5/2}\,dx$ **(06 Marks)**

d. Trace the curve $r = a\sin 3\theta$ **(06 Marks)**

4 a. Choose your answers for the following : **(04 Marks)**

i) Length of one arch of the cyliod $x = a(t-\sin t)$, $y = a(1-\cos t)$ is

A) $-8a$ B) $8a$ C) $\sqrt{8}\,a$ D) $\frac{1}{8a}$

ii) Surface area of revolution about x-axis is s =

A) $2\pi\int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$ B) $\pi\int_a^b\sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$

C) $2\pi\int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dy$ D) $\pi\int_a^b\sqrt{1+\left(\frac{dx}{dy}\right)^2}\ dy$

iii) Area of the cardiode $r = a(1+\cos\theta)$ is

A) $\int_0^{\pi}r^2\,d\theta$ B) $2\int_0^{\pi}r\,d\theta$ C) $\frac{1}{2}\int_0^{\pi/2}r\cos\theta\,d\theta$ D) $2\int_0^{\pi}\cos\theta\,d\theta$

iv) Length of the loop of the curve $x = t^2$, $y = t - t^3/3$ is

A) $2\sqrt{3}$ B) $-4\sqrt{3}$ C) $\frac{1}{4}\sqrt{3}$ D) $4\sqrt{3}$

b. Find the area enclosed by the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$. **(04 Marks)**

c. Find the area of surface of he solid generated when the cardioid $r = a(1 + \cos\theta)$ revolved about the initial line. **(06 Marks)**

d. Prove that $\int_0^{\pi/2} \frac{\log(1 + y\sin^2 x)}{\sin^2 x} = \pi\left[\sqrt{1+y} - 1\right]$ **(06 Marks)**

PART – B

5 a. Choose your answers for the following : **(04 Marks)**

i) Homogeneous differential equation can be reduced to a differential equation by substitution

A) $x + y = v$ B) $y = vx$ C) $xy = v$ D) $x - y = v$.

ii) $(1 + xy)ydx + (1 - xy)xdy = 0$ then I.F. is

A) $2x^2y^2$ B) x^2y^2 C) $\frac{1}{2x^2y^2}$ D) $\frac{2}{x^2y^2}$

iii) The equation $y - 2x = c$ represents the orthogonal trajectories of the family

A) $y = ae^{-2x}$ B) $x^2 + 2y^2 = a^2$ C) $xy = a$ D) $x + 2y = a$

iv) The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ is

A) $\sin\frac{y}{x} = c$ B) $\sin\frac{y}{x} = cx$ C) $\cos\frac{y}{x} = cx$ D) $\cos\frac{y}{x} = c$

b. Solve $(x - y\log y + y\log x)dx + x(\log y - \log x)dx = 0$ **(04 Marks)**

c. Solve $x^3\frac{dy}{dx} - x^2y = -y^4\cos x$ **(06 Marks)**

d. Test for self orthogonality $r^n = a\sin n\theta$. **(06 Marks)**

6 a. Choose your answers for the following : **(04 Marks)**

i) The series $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots\dots\dots$ is

A) Conditionally convergent B) Absolutely convergent

C) Divergent D) None of these.

ii) $\sum\left[1 + \frac{1}{n}\right]^{-n^2}$ is

A) Oscillatory B) Convergent C) Divergent D) Absolutely convergent

iii) By Raabe's test Σu_n is convergent if $\lim_{n\to\infty}\left[\frac{u_n}{u_{n+1}} - 1\right]$

A) Equal to one B) Greater than one C) Less than one D) None of these.

iv) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$ by Lebnitz's test

A) Monotonic decreasing B) Divergence

C) Oscillatory D) Convergency

b. Test the series for convergence, $\frac{3}{4} + \frac{3.6}{4.7} + \frac{3.6.9}{4.7.10} + \dots\dots\dots\dots$ **(04 Marks)**

c. Find the nature of the series $\sum_{n=1}^{\infty}\left(1-\frac{3}{n}\right)^{n^2}$ **(06 Marks)**

d. Test the series $x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+.....$ for absolute convergence. **(06 Marks)**

7 a. Choose your answers for the following : **(04 Marks)**

i) The sum of the direction cosines of a straight line is
A) zero B) one C) constant D) None of these.

ii) The equation of a straight line parallel to the x-axis is given by
A) $\frac{x-a}{1}=\frac{y-b}{1}=\frac{z-c}{1}$ B) $\frac{x-a}{0}=\frac{y-b}{1}=\frac{z-c}{1}$
C) $\frac{x-a}{0}=\frac{y-b}{0}=\frac{z-c}{1}$ D) $\frac{x-a}{1}=\frac{y-b}{0}=\frac{z-c}{0}$

iii) A line makes angles α, β, γ with the co-ordinate axes then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$
A) 1 B) 2 C) 3 D) 0

iv) Three lines are coplanar if
A) They are concurrent
B) A line perpendicular to each other
C) They are concurrent and a line is perpendicular to each of them
D) None of these.

b. Show that the angle between two diagonals of a cube is $\cos^{-1}\frac{1}{3}$ **(04 Marks)**

c. Find the equation of the plane which bisects the line joining (3, 0, 5) and (1, 2, –1) at right angles. **(06 Marks)**

d. Find the shortest distance between the lines and its equations:
$\frac{x-8}{3}=\frac{y+9}{16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. **(06 Marks)**

8 a. Choose your answers for the following : **(04 Marks)**

i) Curl(ϕu) is equal to
A) ϕ grad u + u grad ϕ B) gradϕ.u + ϕdiv u
C) (grad ϕ)×u + ϕ(curl u) D) (ϕ.∇)u + (u.∇)ϕ

ii) If curl F = 0 then the vector F is said to be
A) solenoidal B) Rotational C) Irrotational D) Angular velocity

iii) If $\vec{r}=xi+yj+zk$ then $\nabla.\vec{r}$ is equal to
A) 3 B) 2 C) 1 D) 0

iv) If $F=\nabla(x^3+y^3+z^3-3xyz)$ then curl F is
A) 6(x + y + z) B) (x + y + z) C) 1 D) 0.

b. Find the directional derivative of $\phi=xy^2+yz^3$ at (1, 2, –1) in the direction normal to the surface $x\log z-y^2=-4$ at (–1, 2, 1). **(04 Marks)**

c. Find the constants a and b such that
$\vec{F}=(axy+z^3)i+(3x^2-z)j+(bxz^2-y)k$ is irrotational and find the scalar function such that $F=\nabla\phi$. **(06 Marks)**

d. If $A=2x^2i-3yzj+xz^2k$ and $\phi=2z-x^3y$ find $A=\nabla\phi$ and $(A\times\nabla\phi)$ at (1, –1, 1). **(06 Marks)**

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